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Weak boson scattering in gauge-Higgs unification

Naoyuki Haba,[a] **Yutaka Sakamura**[b] **and Toshifumi Yamashita**[c]

[a] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[b] *RIKEN, Wako, Saitama 351-0198, Japan*
[c] *Department of Physics, Nagoya University, Nagoya 464-8602, Japan*
E-mail: haba@het.phys.sci.osaka-u.ac.jp, sakamura@riken.jp, yamasita@eken.phys.nagoya-u.ac.jp

Abstract: The scattering amplitude for the longitudinal weak bosons is investigated in the SU(3) gauge-Higgs unification as a function of the scattering energy, the Wilson line phase $\theta_{\rm H}$ and the warp factor. The $\theta_{\rm H}$-dependence of the amplitude is quite different in the flat and the warped spacetimes. Generically the amplitude is enhanced for $\theta_{\rm H} = \mathcal{O}(1)$ in the warped case while it is almost independent of $\theta_{\rm H}$ in the flat case. This indicates the tree-level unitarity is violated in the warped case at a lower scale than that in the flat case.

Keywords: Spontaneous Symmetry Breaking, Field Theories in Higher Dimensions, Higgs Physics, Beyond Standard Model

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1 Introduction

The origin of the electroweak symmetry breaking is still a mystery since the Higgs boson has not been discovered yet. Extra dimensions open up new possibility for it. For example, it can occur by nontrivial boundary conditions or the Wilson line phases along the extra dimensions, such as the Higgsless models [1] or the gauge-Higgs unification models [2]–[6]. Since these models are based on higher dimensional gauge theories and are thus nonrenormalizable, they should be interpreted as effective theories of more fundamental theories, which are valid below certain cutoff energy scales. This implies that the tree-level unitarity is violated at some scale, which is identified with the cutoff scale of the model. When we work in a higher dimensional theory, we have to know the cutoff scale of the theory in order to ensure the validity of the perturbative calculation.

The tree-level unitarity is usually discussed by evaluating the scattering amplitudes of the longitudinally polarized weak bosons $W_L^\pm$ and Z_L at tree-level. In the standard model, the Higgs boson plays an important role for the recovery of the tree-level unitarity. If it is sufficiently heavy and decoupled, the scattering amplitudes for $W_L^\pm$ and Z_L grow as E^2 where E is the energy scale of the scattering, and exceed the unitarity bound at some scale

around 1 TeV. This means that the perturbative calculation is no longer reliable above the scale. In the Higgsless models, the tree-level unitarity is recovered by the Kaluza-Klein (KK) modes of the gauge bosons [1] instead of the Higgs boson in the standard model, and the unitarity violation delays up to $\mathcal{O}(10$ TeV) when the compactification scale is assumed to be around 1 TeV.

The situation is more complicated in the gauge-Higgs unification models because they have the Higgs mode (the fluctuation of the Wilson line phase $\theta_{\rm H}$) as well as the KK modes of the gauge bosons, both of which participate in the unitarization of the theory. In these models, both the coupling constants and the KK mass scale depend on $\theta_{\rm H}$ and thus the scattering amplitudes for $W_L^\pm$ and Z_L have nontrivial $\theta_{\rm H}$-dependences. Especially the gauge-Higgs unification in the Randall-Sundrum warped spacetime [7]–[11] is interesting because the WWH and the ZZH couplings (H stands for the Higgs mode) deviate from the standard model values and vanish for some specific values of $\theta_{\rm H}$, such as π or $\pi/2$, depending on the models [10, 11]. For such values of $\theta_{\rm H}$, the Higgs mode cannot participate in the unitarization of the weak boson scattering. Therefore it is important to understand the $\theta_{\rm H}$-dependence of the scattering amplitudes for $W_L^\pm$ and Z_L in order to estimate the cutoff scale of the models from the violation of the tree-level unitarity. This issue is discussed in ref. [12] and some qualitative behaviors of the amplitude are clarified.

In this paper, we investigate various behaviors of the scattering amplitude more quantitatively by numerical calculations. We focus on the process: $W_L^+ + W_L^- \to Z_L + Z_L$ in the gauge-Higgs unification model based on the five-dimensional (5D) SU(3) gauge theory on S^1/Z_2 as the simplest example. Although this model gives the wrong value of the Weinberg angle θ_W and thus is not realistic, it has a lot of common features among the gauge-Higgs unification models. Hence it is a good starting point to understand the behaviors of the amplitudes peculiar to the gauge-Higgs unification. The Wilson line phase $\theta_{\rm H}$, which corresponds to the vacuum expectation value (VEV) of the Higgs field in the standard model, is dynamically determined at one-loop order if the whole matter content of the model is specified. In the following discussion, we do not specify the fermion sector and treat $\theta_{\rm H}$ as a free parameter because we are interested in the tree-level amplitude.

The paper is organized as follows. In section 2, we briefly review the SU(3) gauge-Higgs unification model and provide necessary ingredients to calculate the scattering amplitude for the weak bosons. In section 3, we provide explicit expressions of the scattering amplitudes for the longitudinal weak bosons and for the (would-be) Nambu-Goldstone (NG) bosons, and show their behaviors as functions of E, $\theta_{\rm H}$ and the warp factor. Section 4 is devoted to the summary and discussions. In appendix A, we give definitions and explicit forms of the basis functions used in the text. In appendix B, we derive the 5D propagators of the gauge fields.

2 SU(3) model

2.1 Set-up

We consider the 5D SU(3) gauge theory compactified on S^1/Z_2 as the simplest example of the gauge-Higgs unification. Arbitrary background metric with 4D Poincaré symmetry

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can be written as

$$ds^2 = G_{MN}dx^M dx^N = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{2.1}$$

where $M, N = 0, 1, 2, 3, 4$ are 5D indices and $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. The fundamental region of S^1/Z_2 is $0 \le y \le L$. The function $e^{\sigma(y)}$ is a warp factor, which is normalized as $\sigma(0) = 0$. For example, $\sigma(y) = 0$ in the flat spacetime, and $\sigma(y) = ky$ $(0 \le y \le L)$ in the Randall-Sundrum spacetime [13], where k is the inverse AdS curvature radius.

The 5D gauge field A_M is decomposed as

$$A_M = \sum_{\alpha=1}^{8} A_M^\alpha \frac{\lambda^\alpha}{2}, \tag{2.2}$$

where λ^α are the Gell-Mann matrices. The 5D Lagrangian is

$$\mathcal{L} = \sqrt{-G}\left[-\mathrm{tr}\left\{\frac{1}{2}G^{ML}G^{NP}F_{MN}F_{LP} + \frac{1}{\xi}\left(f_{\mathrm{gf}}\right)^2\right\}\right] + \cdots, \tag{2.3}$$

where $\sqrt{-G} \equiv \sqrt{-\det(G_{MN})} = e^{-4\sigma}$, $F_{MN} \equiv \partial_M A_N - \partial_N A_M - ig_5\left[A_M, A_N\right]$ (g_5 is the 5D gauge coupling), and ξ is a dimensionless parameter. The ellipsis denotes the ghost and the matter sectors, which are irrelevant to the following discussion. The gauge-fixing function f_{gf} is chosen as

$$\begin{aligned} f_{\mathrm{gf}} &= e^{2\sigma}\left\{\eta^{\mu\nu}\partial_\mu A_\nu + \xi\mathcal{D}_y^c\left(e^{-2\sigma}A_y\right)\right\}, \\ \mathcal{D}_y^c A_M &\equiv \partial_y A_M - ig_5\left[A_y^{\mathrm{bg}}, A_M\right], \end{aligned} \tag{2.4}$$

where $A_y^{\mathrm{bg}}(y)$ is the classical background of $A_y(x, y)$.

The boundary conditions for the gauge field is written as

$$\begin{aligned} A_\mu(x, -y) &= P_0 A_\mu(x, y)P_0^{-1}, & A_\mu(x, L+y) &= P_L A_\mu(x, L-y)P_L^{-1}, \\ A_y(x, -y) &= -P_0 A_y(x, y)P_0^{-1}, & A_y(x, L+y) &= -P_L A_y(x, L-y)P_L^{-1}, \end{aligned} \tag{2.5}$$

where P_0 and P_L are unitary matrices satisfying the relation $P_0^2 = P_L^2 = 1$. By choosing them as $P_0 = P_L = \mathrm{diag}(-1, -1, 1)$, the Z_2-parity eigenvalues (P_0, P_L) of the gauge fields become

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, \qquad A_y = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}. \tag{2.6}$$

Note that only $(+,+)$ fields can have zero-modes when perturbation theory is developed around the trivial configuration $A_M = 0$. Thus the SU(3) gauge symmetry is broken to $\mathrm{SU}(2) \times \mathrm{U}(1)$ at tree-level. The zero-modes of A_y form an SU(2)-doublet 4D scalar $(A_y^4 + iA_y^5, A_y^6 + iA_y^7)$, which plays a role of the Higgs doublet in the standard model whose VEV breaks $\mathrm{SU}(2) \times \mathrm{U}(1)$ to $\mathrm{U}(1)_{\mathrm{EM}}$. They yield non-Abelian Aharonov-Bohm phases (Wilson line phases) when integrated along the fifth dimension. By using the residual $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry, we can always push the nonvanishing VEV into A_y^7. Then the Wilson line phase θ_{H} is given by

$$\theta_{\mathrm{H}} = g_5 \int_0^L dy\ A_y^{\mathrm{bg}\,7}(y). \tag{2.7}$$

2.2 Mode expansion

The mode expansion of the 5D gauge fields is performed in a conventional way (see ref. [8], for example). For the following discussion, it is convenient to move to the momentum representation for the 4D part while remain the coordinate representation for the fifth dimension. Then the 5D gauge fields are expanded into the KK modes as

$$\begin{aligned}\tilde{A}^{\alpha}_{\mu}(p,y) &= \sum_n u^{\alpha}_n(y) A^{(n)}_{\mu}(p) + \sum_n w^{\alpha}_n(y) p_{\mu} A^{(n)}_{\mathrm{S}}(p), \\ \tilde{A}^{\alpha}_{y}(p,y) &= \sum_n v^{\alpha}_n(y) \varphi^{(n)}(p).\end{aligned} \tag{2.8}$$

Here we have moved to the Scherk-Schwarz basis, in which $\tilde{A}^{\mathrm{bg}}_y = 0$. It is related to the original basis by the gauge transformation,

$$\tilde{A}_M = \Omega A_M \Omega^{-1} - \frac{i}{g_5} (\partial_M \Omega)\, \Omega^{-1}, \tag{2.9}$$

with

$$\Omega(y) \equiv \mathcal{P} \exp\left\{ -i g_5 \int_0^y dy'\ A^{\mathrm{bg}\,7}_y(y') \cdot \frac{\lambda^7}{2} \right\}. \tag{2.10}$$

The symbol $\mathcal{P}$ stands for the path ordered operator from left to right. Notice that $\tilde{A}^{\alpha}_{\mu}(p,y)$ are decomposed into two parts, according to their polarization. In the above expression, $A^{(n)}_{\mu}(p)$ are polarized as $p^{\mu} A^{(n)}_{\mu}(p) = 0$ and include the transverse and the longitudinal modes, which are physical for the massive modes. On the other hand, $A^{(n)}_{\mathrm{S}}(p)$ are unphysical scalar modes. The gauge-scalar modes $\varphi^{(n)}(p)$ are also unphysical besides the zero-mode.

The mode functions $w^{\alpha}_n(y)$ and $v^{\alpha}_n(y)$ are related to each other by

$$\begin{aligned} v^{\alpha}_n(y) &= \frac{d}{dy} w^{\alpha}_n(y), \\ w^{\alpha}_n(y) &= -\frac{\xi}{\tilde{m}_n^2} \frac{d}{dy} \left\{ e^{-2\sigma(y)} v^{\alpha}_n(y) \right\}, \end{aligned} \tag{2.11}$$

where $\tilde{m}_n$ are common mass eigenvalues for $A^{(n)}_{\mathrm{S}}$ and $\varphi^{(n)}$. These relations hold irrespective of the value of the gauge parameter ξ. When $\xi = 1$, they are further related to $u^{\alpha}_n(y)$ as

$$u^{\alpha}_n(y) = m_n w^{\alpha}_n(y), \qquad m_n = \tilde{m}_n, \tag{2.12}$$

where m_n are mass eigenvalues for $A^{(n)}_{\mu}$.

According to the transformation properties under the unbroken $\mathrm{U}(1)_{\mathrm{EM}}$ symmetry and the rotation by a constant matrix $\Omega(L)$, the gauge fields are classified into the charged sector $(A^{1\pm i2}_M, A^{4\pm i5}_M) \equiv (A^1_M \pm i A^2_M, A^4_M \pm i A^5_M)/\sqrt{2}$ and the neutral sectors $(A^{3'}_M, A^6_M)$, A^7_M and $A^{8'}_M$, where

$$\begin{pmatrix} A^{3'}_M \\ A^{8'}_M \end{pmatrix} \equiv \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} A^3_M \\ A^8_M \end{pmatrix}. \tag{2.13}$$

The W, Z bosons and the photon are identified with the lowest modes in the $(A^{1\pm i2}_{\mu}, A^{4\pm i5}_{\mu})$-, the $(A^{3'}_{\mu}, A^6_{\mu})$- and the $A^{8'}_{\mu}$-sectors, respectively.

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The mode functions for the W boson are calculated as

$$u_W^1(y) \equiv u_0^{1+i2}(y) = N_W c_\theta S_0(L, m_W) C_0(y, m_W),$$
$$u_W^4(y) \equiv u_0^{4+i5}(y) = -N_W s_\theta C_0(L, m_W) S_0(y, m_W), \tag{2.14}$$

where $c_\theta \equiv \cos(\theta_{\rm H}/2)$, $s_\theta \equiv \sin(\theta_{\rm H}/2)$, and $C_0(y,m)$, $S_0(y,m)$ are the basis functions defined in appendix A. The mode functions for the Z boson are

$$u_Z^{3'}(y) \equiv u_0^{3'}(y) = N_Z \cos\theta_{\rm H} S_0(L, m_Z) C_0(y, m_Z),$$
$$u_Z^6(y) \equiv u_0^6(y) = -N_Z \sin\theta_{\rm H} C_0(L, m_Z) S_0(y, m_Z). \tag{2.15}$$

Here the normalization constants N_W and N_Z are determined by

$$\int_0^L dy \ \left\{(u_W^1)^2 + (u_W^4)^2\right\} = 1, \qquad \int_0^L dy \ \left\{(u_Z^{3'})^2 + (u_Z^6)^2\right\} = 1, \tag{2.16}$$

and the W and the Z boson masses m_W and m_Z are the lowest solutions of

$$\frac{1}{m_W}\left\{C_0'(L, m_W) S_0(L, m_W) + m_W e^{\sigma(L)} s_\theta^2\right\} = 0,$$
$$\frac{1}{m_Z}\left\{C_0'(L, m_Z) S_0(L, m_Z) + m_Z e^{\sigma(L)} \sin^2\theta_{\rm H}\right\} = 0, \tag{2.17}$$

respectively. The prime denotes the derivative for y. In the flat spacetime, for example, they are written for $0 < \theta_{\rm H} < \pi$ as

$$m_W = \frac{\theta_{\rm H}}{2L}, \qquad m_Z = \frac{|\pi - |\pi - 2\theta_{\rm H}||}{2L}. \tag{2.18}$$

For the corresponding gauge-scalar modes, the mode functions $v_W^{1,4}(y)$, $v_Z^{3',6}(y)$ and their masses are obtained similarly.

2.3 5D propagators

For the purpose of calculating the scattering amplitude, it is convenient to use the 5D propagators defined in a mixed momentum/position representation [14]. It describes the propagation of the entire KK tower of excitations carrying the 4D momentum p between two points y and y' in the extra dimension. This approach has an advantage that we need not explicitly calculate the mass eigenvalues for the modes propagating in the internal lines of the Feynmann diagrams nor sum over contributions from the infinite number of KK modes.[1] The explicit forms of the 5D propagators can be obtained by using the formula (B.10). For the charged sector $(A_\mu^{1+i2}, A_\mu^{4+i5})$, the 5D propagator is calculated as

$$G_{\rm T<}(y,y') = \frac{e^{2\sigma(L)}}{\det_{(1,4)}\mathcal{W}} \begin{pmatrix} |p|\, e^{\sigma(L)} C_0(y) S_0(L) C_L(y') & \\ & -S_0(y) C_0'(L) S_L(y') \end{pmatrix}$$
$$- \frac{s_\theta\, |p|\, e^{3\sigma(L)}}{\det_{(1,4)}\mathcal{W}} \begin{pmatrix} s_\theta C_0(y) S_0(y') & c_\theta C_0(y) S_0(y') \\ c_\theta S_0(y) C_0(y') & -s_\theta S_0(y) C_0(y') \end{pmatrix},$$
$$G_{\rm T>}(y,y') = \left\{G_{\rm T<}(y',y)\right\}^t, \tag{2.19}$$

[1] This approach is also useful for models with continuum spectra [15].

where $|p| \equiv \sqrt{-p^2}$, and $\det_{(1,4)}$ is the determinant of $\mathcal{W}$ defined in (B.11) restricted to the $(A_\mu^{1+i2}, A_\mu^{4+i5})$-sector and

$$\det_{(1,4)}\mathcal{W} = -|p|\, e^{\sigma(L)} \left\{ C_0'(L) S_0(L) + |p|\, e^{\sigma(L)} s_\theta^2 \right\}. \tag{2.20}$$

In the above expressions, we have omitted $|p|$ from the arguments. For the $(A_\mu^{3'}, A_\mu^6)$-sector, the propagator is obtained from the above expressions by replacing $\theta_{\rm H}/2$ with $\theta_{\rm H}$. The propagators in the other sectors are calculated as

$$\begin{aligned} G_{\rm T<}^{77}(y, y', |p|) &= e^{\sigma(L)} \frac{S_0(y,|p|) S_L(y',|p|)}{|p|\, S_0(L,|p|)}, \\ G_{\rm T<}^{8'8'}(y, y', |p|) &= -e^{2\sigma(L)} \frac{C_0(y,|p|) C_L(y',|p|)}{C_0'(L,|p|)}. \end{aligned} \tag{2.21}$$

3 Weak boson scattering

Now we consider the scattering of the weak bosons. The two-body scattering amplitudes are functions of the total energy E and the scattering angle ϕ in the center-of-mass frame. For the elastic scattering of the W bosons, there is an infrared singularity at $\phi = 0$ due to the t-channel diagram exchanging the massless photon. Although such a singularity will be cancelled when the soft-photon emissions and higher loop corrections are taken into account, it involves some technical difficulties to obtain a finite value of the amplitude for the forward scattering. In order to avoid such difficulties, we consider the scattering process: $W_L^+(p_1) + W_L^-(p_2) \to Z_L(p_3) + Z_L(p_4)$ in the following. The subscript L denotes the longitudinal polarization.

3.1 Scattering amplitude

As mentioned in section 2.3, the scattering amplitudes are easily calculated by utilizing the 5D propagators. The scattering amplitude $\mathcal{A}$ is expressed by

$$\mathcal{A} = \mathcal{A}^{\rm C} + \mathcal{A}^{\rm V} + \mathcal{A}^{\rm S}, \tag{3.1}$$

where $\mathcal{A}^{\rm C}$, $\mathcal{A}^{\rm V}$ and $\mathcal{A}^{\rm S}$ are contributions from the contact interaction, the exchange of the vector modes and that of the gauge-scalar modes, respectively. They are given by

$$\begin{aligned} \mathcal{A}^{\rm C} &= -\frac{ig_5^2}{4} \int_0^L dy \ \left\{ (u_W^1)^2 + (u_W^4)^2 \right\} \left\{ (u_Z^{3'})^2 + (u_Z^6)^2 \right\} \\ &\quad \times \left\{ 2(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3^* \cdot \varepsilon_4^*) - (\varepsilon_1 \cdot \varepsilon_3^*)(\varepsilon_2 \cdot \varepsilon_4^*) - (\varepsilon_1 \cdot \varepsilon_4^*)(\varepsilon_2 \cdot \varepsilon_3^*) \right\}, \end{aligned} \tag{3.2}$$

$$\begin{aligned} \mathcal{A}^{\rm V} &= ig_5^2 \sum_{\alpha,\beta=1,4} \int_0^L dy \int_0^L dy' \ U_{\rm WZ}^\alpha(y) G_{\rm T}^{\alpha\beta}(y, y', |p_{13}|) U_{\rm WZ}^\beta(y') P_{1324} \\ &\quad + ig_5^2 \sum_{\alpha,\beta=1,4} \int_0^L dy \int_0^L dy' \ U_{\rm WZ}^\alpha(y) G_{\rm T}^{\alpha\beta}(y, y', |p_{14}|) U_{\rm WZ}^\beta(y') P_{1423}, \end{aligned} \tag{3.3}$$

$$\begin{aligned} \mathcal{A}^{\rm S} &= ig_5^2 \int_0^L dy \ Y_{\rm WW}^7(y) Y_{\rm ZZ}^7(y) \frac{(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3^* \cdot \varepsilon_4^*)}{p_{12}^2} \\ &\quad + ig_5^2 \sum_{\alpha=1,4} \int_0^L dy \ Y_{\rm WZ}^\alpha(y) Y_{\rm WZ}^\alpha(y) \left\{ \frac{(\varepsilon_1 \cdot \varepsilon_3^*)(\varepsilon_2 \cdot \varepsilon_4^*)}{p_{13}^2} + \frac{(\varepsilon_1 \cdot \varepsilon_4^*)(\varepsilon_2 \cdot \varepsilon_3^*)}{p_{14}^2} \right\}, \end{aligned} \tag{3.4}$$

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where ε_i $(i = 1, 2, 3, 4)$ are the polarization vectors, $p_{12} \equiv p_1 + p_2$, $p_{13} \equiv p_1 - p_3$, $p_{14} \equiv p_1 - p_4$,

$$
\begin{aligned}
P_{1324} \equiv & \{2(p_1 \cdot \varepsilon_3^*)\varepsilon_1 + 2(p_3 \cdot \varepsilon_1)\varepsilon_3^* - (\varepsilon_1 \cdot \varepsilon_3^*)(p_1 + p_3)\}^\mu \left(\eta_{\mu\nu} - \frac{p_{13\mu}p_{13\nu}}{p_{13}^2}\right) \\
& \times \{2(p_2 \cdot \varepsilon_4^*)\varepsilon_2 + 2(p_4 \cdot \varepsilon_2)\varepsilon_4^* - (\varepsilon_2 \cdot \varepsilon_4^*)(p_2 + p_4)\}^\nu , \\
P_{1423} \equiv & \{2(p_1 \cdot \varepsilon_4^*)\varepsilon_1 + 2(p_4 \cdot \varepsilon_1)\varepsilon_4^* - (\varepsilon_1 \cdot \varepsilon_4^*)(p_1 + p_4)\}^\mu \left(\eta_{\mu\nu} - \frac{p_{14\mu}p_{14\nu}}{p_{14}^2}\right) \\
& \times \{2(p_2 \cdot \varepsilon_3^*)\varepsilon_2 + 2(p_3 \cdot \varepsilon_2)\varepsilon_3^* - (\varepsilon_2 \cdot \varepsilon_3^*)(p_2 + p_3)\}^\nu ,
\end{aligned} \tag{3.5}
$$

and the functions in the integrands are defined as

$$
\begin{aligned}
U^1_{\rm WZ} &= \frac{1}{2}\left(u^1_W u^{3'}_Z + u^4_W u^6_Z\right), & U^4_{\rm WZ} &= \frac{1}{2}\left(u^1_W u^6_Z - u^4_W u^{3'}_Z\right), \\
Y^7_{\rm WW} &= e^{-2\sigma}\left\{(u^1_W)' u^4_W - u^1_W (u^4_W)'\right\}, & Y^7_{\rm ZZ} &= 2e^{-2\sigma}\left\{(u^{3'}_Z)' u^6_Z - u^{3'}_Z (u^6_Z)'\right\}, \\
Y^1_{\rm WZ} &= \frac{e^{-2\sigma}}{2}\left\{(u^1_W)' u^{3'}_Z - u^1_W (u^{3'}_Z)' + (u^4_W)' u^6_Z - u^4_W (u^6_Z)'\right\}, \\
Y^4_{\rm WZ} &= \frac{e^{-2\sigma}}{2}\left\{(u^1_W)' u^6_Z - u^1_W (u^6_Z)' - (u^4_W)' u^{3'}_Z + u^4_W (u^{3'}_Z)'\right\}.
\end{aligned} \tag{3.6}
$$

Here we have used the relation $p_i \cdot \varepsilon_i(p_i) = 0$ $(i = 1, 2, 3, 4)$.

The first and the second lines in (3.3) correspond to the t-channel and the u-channel diagrams exchanging the W boson and its KK modes, respectively. The above expression of the amplitude is a result of a cancellation between the gauge-dependent part $G_{\rm S}(y, y', |p|)$ in the propagator of the vector modes and the gauge-scalar propagator $G_{yy}(y, y', |p|)$. This cancellation occurs due to the relation (B.16) and makes the resultant amplitude gauge-independent. The contribution $\mathcal{A}^{\rm S}$ is a remnant of the cancellation. The first line in (3.4) corresponds to the s-channel diagram exchanging the neutral gauge-scalar mode (i.e., the Higgs boson) while the second line represents the contributions from the t-channel and the u-channel exchanges of the charged gauge-scalar modes. Notice that the Higgs boson is massless at tree-level in the gauge-Higgs unification. It acquires a nonzero mass at one-loop order.

The gauge invariance of the theory ensures the equivalence theorem [16], which states that the scattering of the longitudinally polarized vector bosons is equivalent to that of the (would-be) NG bosons eaten by the gauge bosons. In 5D models, the gauge-scalar modes $\varphi^{(n)}$ coming from A_y play the role of the NG bosons in the equivalence theorem [1, 17].[2] Namely, the following relation holds for the longitudinal vector modes $A_L^{(n)}$.

$$
T(A_L^{(n_1)}, \cdots, A_L^{(n_l)}; \Phi) = C_l T(i\varphi^{(n_1)}, \cdots, i\varphi^{(n_l)}; \Phi) + \mathcal{O}\left(\frac{M^2}{E^2}\right), \tag{3.7}
$$

where all external lines are directed inwards, Φ denotes any possible amputated external physical fields, such as the transverse gauge boson, and M is the heaviest mass among the

[2] When there exist additional bulk Higgs fields that break the gauge symmetry, the NG bosons become mixture of modes from the bulk Higgs fields and A_y.

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external lines. A constant C_l is gauge-dependent, but $C_l = 1$ at tree-level.[3] The correction term is $\mathcal{O}(M^2/E^2)$ because of the 5D gauge invariance (see ref. [19], for example). Eq. (3.7) is useful to discuss the high-energy behavior of the scattering amplitude $\mathcal{A}$ because the corresponding NG boson amplitude does not have $\mathcal{O}(E^4)$ contributions,[4] which makes it easier to numerically calculate the amplitude thanks to the absence of cancellations between large numbers.

The scattering amplitude for the corresponding NG bosons comes only from the diagrams exchanging the vector modes.

$$\begin{aligned}\mathcal{B} = & ig_5^2 \sum_{\alpha,\beta=1,4} \int_0^L dy \int_0^L dy' \; V_{\rm WZ}^{\alpha}(y)(p_1+p_3)^{\mu} G_{\mu\nu}^{\alpha\beta}(p_{13},y,y')(p_2+p_4)^{\nu} V_{\rm WZ}^{\beta}(y') \\ & + ig_5^2 \sum_{\alpha,\beta=1,4} \int_0^L dy \int_0^L dy' \; V_{\rm WZ}^{\alpha}(y)(p_1+p_4)^{\mu} G_{\mu\nu}^{\alpha\beta}(p_{14},y,y')(p_2+p_3)^{\nu} V_{\rm WZ}^{\beta}(y'), \end{aligned} \tag{3.8}$$

where

$$\begin{aligned} V_{\rm WZ}^1 &\equiv \frac{e^{-2\sigma}}{2}\left(v_W^1 v_Z^{3'} + v_W^4 v_Z^6\right), \\ V_{\rm WZ}^4 &\equiv \frac{e^{-2\sigma}}{2}\left(v_W^1 z_Z^6 - v_W^4 v_Z^{3'}\right). \end{aligned} \tag{3.9}$$

3.2 Various behaviors of the amplitudes

Here we show various behaviors of the scattering amplitudes given in the previous subsection. For the numerical calculation, we choose the gauge parameter as $\xi = 1$, the 4D gauge coupling $g_4 \equiv g_5/\sqrt{L}$ as $g_4^2 = 0.1$, and take the W boson mass m_W as an input parameter. Then the size of the extra dimension L becomes $\theta_{\rm H}$-dependent after fixing m_W. (See eq. (2.18), for example.) The KK mass scale $m_{\rm KK} \equiv \pi k/(e^{kL}-1)$ is also $\theta_{\rm H}$-dependent. Thus the amplitudes are functions of the center-of-mass energy E, the Wilson line phase $\theta_{\rm H}$ and the warp factor e^{kL}. The physical amplitude $\mathcal{A}$ is of course gauge-independent, and the ξ-dependence of the NG boson amplitude $\mathcal{B}$ is small in high-energy region as can be seen from (3.7). The 4 momenta and the polarization vectors of the initial and final states are parameterized as in table 1. There, $p_W \equiv \sqrt{E^2/4 - m_W^2}$, $p_Z \equiv \sqrt{E^2/4 - m_Z^2}$, and ϕ is the scattering angle in the center-of-mass frame. Notice that the amplitudes $\mathcal{A}$ in (3.1) and $\mathcal{B}$ in (3.8) are symmetric under $\phi \leftrightarrow \pi - \phi$.

3.2.1 Non-forward scattering

First we consider a non-forward (and non-backward) scattering. We choose the scattering angle as $\phi = \pi/3$ in the following. Figure 1 shows the energy dependence of the scattering amplitudes. The solid and the dashed lines represent the scattering amplitudes for the vector bosons $\mathcal{A}$ and for the NG bosons $\mathcal{B}$, respectively. We can explicitly see that the equivalence theorem holds both in the flat and the warped cases, and $|\mathcal{B}| - |\mathcal{A}| = \mathcal{O}(m_W^2/E^2)$.

[3] We can also take a gauge where $C_l = 1$ at all orders of the perturbative expansion [18].

[4] For the non-forward (non-backward) scattering, $\mathcal{O}(E^2)$ contributions are also absent.

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$p_1 = (E/2, 0, 0, p_W)$	$\varepsilon_1(p_1) = (p_W, 0, 0, E/2)/m_W$
$p_2 = (E/2, 0, 0, -p_W)$	$\varepsilon_2(p_2) = (p_W, 0, 0, -E/2)/m_W$
$p_3 = (E/2, p_Z \sin\phi, 0, p_Z \cos\phi)$	$\varepsilon_3(p_3) = (p_Z, (E/2)\sin\phi, 0, (E/2)\cos\phi)/m_Z$
$p_4 = (E/2, -p_Z \sin\phi, 0, -p_Z \cos\phi)$	$\varepsilon_4(p_4) = (p_Z, -(E/2)\sin\phi, 0, -(E/2)\cos\phi)/m_Z$

Table 1. The 4 momenta and the polarization vectors of the initial and the final states for $W_L^+(p_1) + W_L^-(p_2) \to Z_L(p_3) + Z_L(p_4)$. E is the center-of-mass energy, $p_W \equiv \sqrt{E^2/4 - m_W^2}$, $p_Z \equiv \sqrt{E^2/4 - m_Z^2}$, and ϕ is the scattering angle in the center-of-mass frame.

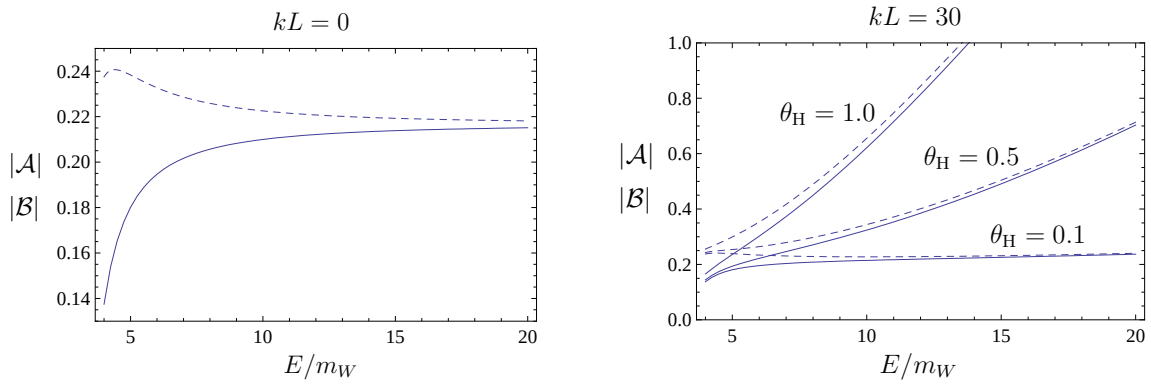

Figure 1. The energy dependence of the amplitudes for $W_L^+ + W_L^- \to Z_L + Z_L$. The solid lines represent the vector boson scattering $\mathcal{A}$, and the dashed lines are the NG boson scattering $\mathcal{B}$. The scattering angle is chosen as $\phi = \pi/3$. In the flat case (the left figure), the amplitudes are independent of the Wilson line phase $\theta_{\rm H}$ for $0 < \theta_{\rm H} < \pi/2$.

In the flat case ($kL = 0$), the amplitude $\mathcal{A}$ is independent of the Wilson line phase $\theta_{\rm H}$ in the range of $0 < \theta_{\rm H} < \pi/2$. It approaches to a constant value at high energies. In the case of the warped geometry, on the other hand, The amplitude has a large $\theta_{\rm H}$-dependence and increase as E^2. It grows faster for larger $\theta_{\rm H}$.

These behaviors reflect the $\theta_{\rm H}$-dependences of the coupling constants among the gauge and the Higgs modes and of the KK mass scale $m_{\rm KK}$. Before explaining the behaviors of the amplitude, let us see the energy dependence of $\mathcal{B}$ again by rescaling the unit of the horizontal axes to $m_{\rm KK}$. (Figure 2) Then we can see that the amplitude approaches to constant values at sufficiently high energies even in the warped case. The constant values vary depending on the warp factor, and are larger than the value in the flat case by a factor kL for $kL \gtrsim \mathcal{O}(1)$. The $\theta_{\rm H}$-dependence we have seen in the right plot of figure 1 now almost disappears in the unit of $m_{\rm KK}$. It is cancelled by the $\theta_{\rm H}$-dependence of $m_{\rm KK}$ (see the beginning of section 3.2). The apparent $\theta_{\rm H}$-dependence of the plots in the flat case stems from the $\theta_{\rm H}$-dependence of $m_{\rm KK}$.

Now we will interpret the above behaviors of the amplitude. First of all, we should notice that the model reduces to the "standard model" (SM), in which the Weinberg angle is $\sin^2\theta_W = 3/4$ and the Higgs boson is massless, when $\theta_{\rm H} \ll 1$ irrespective of the 5D geometry. Every coupling constant in the gauge-Higgs sector takes almost the SM value and the KK modes are heavy enough to decouple. Thus the amplitude takes the same

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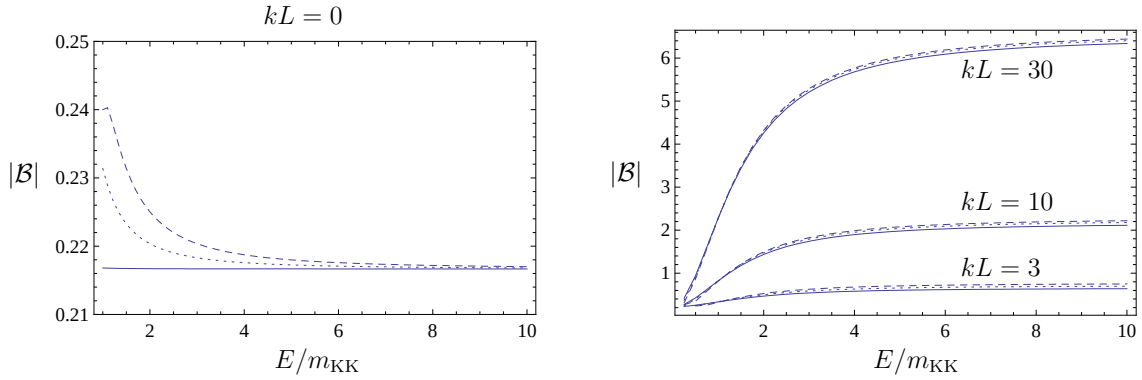

Figure 2. The energy dependence of the amplitude in the unit of $m_{\rm KK}$. The solid, dotted, dashed lines correspond to $\theta_{\rm H} = 0.1, 1.0, 1.5$, respectively.

value as SM up to the energy scale where the KK modes start to propagate, i.e., $m_{\rm KK}$. The amplitude takes an almost constant value at $E \gtrsim \mathcal{O}(10 m_W)$ in this case.

When $\theta_{\rm H}$ is not small, the coupling constants relevant to the amplitude deviate from the SM value. In the warped case, the WWH and the ZZH couplings become smaller than the SM values by $\cos(\theta_{\rm H}/2)$ and $\cos\theta_{\rm H}$ respectively, while the $WWZZ$ and the WWZ couplings are almost unchanged [10, 11]. Thus $\mathcal{O}(E^2)$ contributions miss to be cancelled among the low-lying modes and the amplitude grows in the low-energy region. For larger value of $\theta_{\rm H}$ (up to $\pi/2$), the deviation of the couplings are larger and then the amplitude grows faster. (See the right figure of figure 1.) This remaining $\mathcal{O}(E^2)$ contribution is eventually cancelled by contributions from the KK modes. Namely, the amplitude ceases to increase and approaches to a constant value when the KK modes start to propagate.

The flat spacetime is a special case. As we mentioned, the amplitude becomes almost constant at $E \gtrsim \mathcal{O}(10 m_W)$ when $\theta_{\rm H} \ll 1$. For larger values of $\theta_{\rm H}$, the $WWZZ$ and the WWZ couplings slightly deviate from the SM values because of the nontrivial y-dependences of the mode functions for the W and the Z bosons [11], while the WWH and the ZZH couplings are now unchanged. Then the $\mathcal{O}(E^2)$ contributions fail to be cancelled among the low-lying modes, just like in the warped case. However the contribution from the KK-modes *completely* cancel this $\mathcal{O}(E^2)$, and the amplitude results in unchanged from the $\theta_{\rm H} \ll 1$ case. Namely the effect of the $\theta_{\rm H}$-dependence of the $WWZZ$ and WWZ couplings and that of the KK mass spectrum are completely cancelled and the amplitude becomes $\theta_{\rm H}$-independent for $0 < \theta_{\rm H} \leq \pi/2$ in the flat case. In the range of $\pi/2 < \theta_{\rm H} < \pi$, the amplitude has a nontrivial $\theta_{\rm H}$-dependence. This stems from the fact that the relation $m_Z/m_W = 2$ no longer holds (see eq. (2.18)) and m_Z also has a nontrivial $\theta_{\rm H}$-dependence in this region.

3.2.2 Forward scattering

Next we consider the forward scattering, i.e., $\phi = 0$. In this case, an $\mathcal{O}(E^2)$ contribution remains and the amplitude monotonically increases even above $m_{\rm KK}$. This is because the power counting of E for the amplitude changes around $\phi = 0$. For example, the brace part

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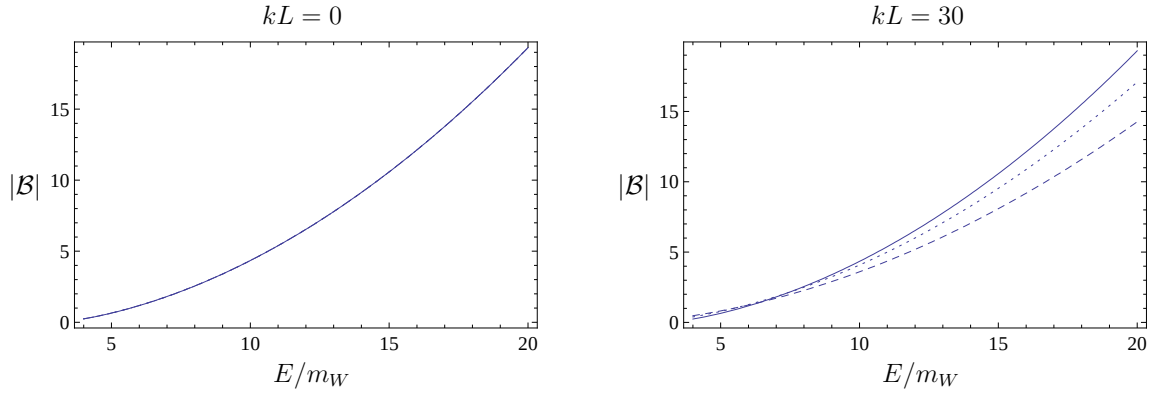

Figure 3. The energy dependence of the amplitude at $\phi = 0$. The solid, dotted, dashed lines correspond to $\theta_{\rm H} = 0.1, 1.0, 1.5$, respectively.

in (3.4) is expanded (for nonzero $\sin\phi$) as

$$\begin{aligned} A_{tu} &\equiv \frac{(\varepsilon_1 \cdot \varepsilon_3^*)(\varepsilon_2 \cdot \varepsilon_4^*)}{p_{13}^2} + \frac{(\varepsilon_1 \cdot \varepsilon_4^*)(\varepsilon_2 \cdot \varepsilon_3^*)}{p_{14}^2} \\ &= \frac{E^2}{4m_W^2 m_Z^2} - \frac{m_W^2 + m_Z^2}{2m_W^2 m_Z^2} + \frac{2m_W^2 m_Z^2 + (m_W^4 + m_Z^4)\cos(2\phi)}{m_W^2 m_Z^2 E^2 \sin^2\phi} + \mathcal{O}(E^{-4}). \end{aligned} \tag{3.10}$$

This means that the expansion becomes invalid when $\sin\phi \lesssim \mathcal{O}(m_W/E)$. At $\phi = 0$, this quantity reduces to

$$A_{tu} = \frac{(m_W^4 + m_Z^4)E^2}{2m_W^2 m_Z^2 (m_Z^2 - m_W^2)^2} - \frac{2(m_W^2 + m_Z^2)}{(m_Z^2 - m_W^2)^2}, \tag{3.11}$$

and the leading term for the high energy expansion changes. Therefore an $\mathcal{O}(E^2)$ contribution is left in the total amplitude. Similar behavior of the amplitude is observed also in the standard model. Figure 3 shows the energy dependence of the forward scattering amplitude. We can see that the amplitude grows as E^2 in any cases. In the flat case, the amplitude does not have the $\theta_{\rm H}$-dependence again. In the warped case, it varies for different values of $\theta_{\rm H}$. For small values of $\theta_{\rm H}$, the amplitude has little dependence on the warp factor and takes almost the same value as the flat case. For larger values of $\theta_{\rm H}$, it becomes smaller in contrast to the non-forward scattering.

3.2.3 S-wave amplitude

The conventional bound for the tree-level unitarity is given by[5]

$$|a_0| \leq 1, \tag{3.12}$$

where a_0 is the s-wave amplitude defined as

$$a_0(E) \equiv \frac{1}{32\pi} \int_{-1}^{1} d(\cos\phi)\ \mathcal{A}(E, \cos\phi). \tag{3.13}$$

[5]More restrictive unitarity condition is proposed in ref. [20].

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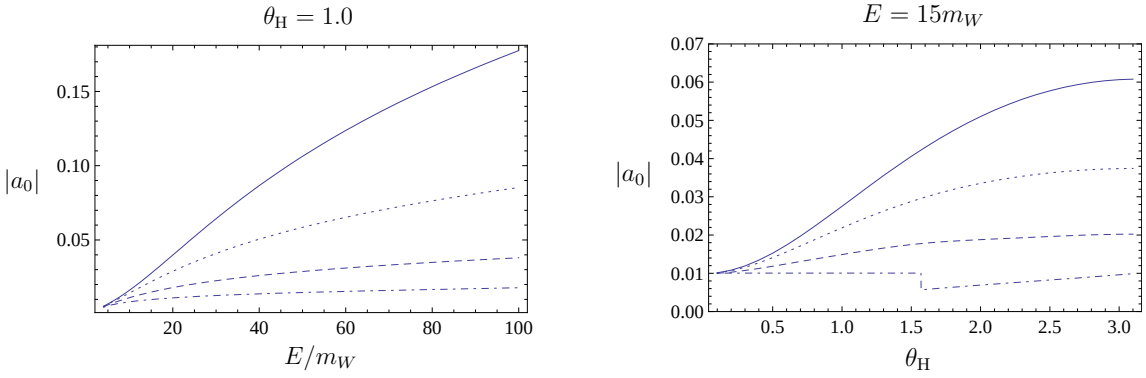

Figure 4. The s-wave amplitude as functions of E and θ_H. The solid, dotted, dashed, dotdashed lines correspond to $kL = 30, 10, 3, 0$, respectively.

Hence we now estimate the s-wave amplitude. As we mentioned above, the integrand grows as E^2 in the region $1 - |\cos\phi| \lesssim \mathcal{O}(m_W^2/E^2)$ while it approaches to a constant for large E in the other region of $\cos\phi$. Therefore $a_0(E)$ behaves as $\mathcal{O}(E^0)$ at high energies. In fact, it grows logarithmically in high-enery region as shown in the left plot of figure 4. The right plot shows the θ_H-dependence of $a_0(15m_W)$. We can see from these plots that the amplitude becomes larger for larger warp factor and larger $\sin(\theta_H/2)$.[6] In the flat limit, it decreases and has only a small θ_H-dependence. In fact, it is independent of θ_H for $0 < \theta_H \leq \pi/2$. The small θ_H-dependence for $\pi/2 < \theta_H < \pi$ originates from the fact that the relation $m_Z/m_W = 2$ no longer holds and m_Z becomes θ_H-dependent, which is peculiar to the SU(3) model. Thus the details of the small θ_H-dependence in the flat case is model-dependent.

In the above calculation, we have chosen the value of the gauge coupling as $g_4^2 = 0.1$. Since the tree-level amplitude is proportional to g_4^2, it becomes four times larger if we take g_4 as the weak gauge coupling in the standard model. In this case, a scale Λ determined by $a_0(\Lambda) = 1$ is estimated as $\Lambda \sim 1500m_W$ for $\theta_H = 0.1$ and $\Lambda \sim 150m_W$ for $\theta_H = 1.5$ when $kL = 30$, for example. This suggests that the perturbative unitarity will be violated at a lower scale for larger $\sin(\theta_H/2)$. In order to estimate the unitarity bound, we have to consider other scattering processes and sum up all the possible final states including the KK states. Thus the real cut-off scale $\Lambda_{\rm cut}$ is expected as much lower scale than the above values of Λ. In particular, in the latter example where $m_{\rm KK} \simeq 18m_W$ and the Higgs mode hardly contributes to the unitarization, it is expected that $\Lambda_{\rm cut}$ becomes around 1 TeV as in the standard model without the Higgs field.

4 Summary and discussions

We have investigated the weak boson scattering in the gauge-Higgs unification, focusing on the dependence of the amplitude on the scattering energy E, the Wilson line phase θ_H and the warp factor e^{kL}. In this paper we consider a process: $W_L^+ + W_L^- \to Z_L + Z_L$ in the SU(3) model as the simplest example.

The 5D propagators are useful to calculate the scattering amplitudes because we need not explicitly calculate the KK mass spectra nor perform the infinite summation over the

[6] Notice that the period of θ_H is 2π. (See the first equation in eq. (2.17).)

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KK modes propagating in the internal lines. We have numerically checked the equivalence theorem between the amplitudes for the longitudinal vector bosons and the (would-be) NG bosons. The correction term is read off as $\mathcal{O}(m_W^2/E^2)$.

The amplitude behaves differently in the flat and the warped spacetimes. It is independent of $\theta_{\rm H}$ in the flat case, while a nontrivial $\theta_{\rm H}$-dependence comes out in the warped case. These behaviors come from the $\theta_{\rm H}$-dependences of the coupling constants among the gauge and the Higgs modes and of the KK mass scale $m_{\rm KK}$ in the case that the W boson mass m_W is fixed as an input parameter (see the beginning of section 3.2). For the non-forward (and non-backward) scattering, the amplitude approaches to a constant at high energies in both cases, but the asymptotic constant value is enhanced by a factor kL in the warped case ($kL \gtrsim \mathcal{O}(1)$), comparing to that in the flat case. On the other hand, the forward (backward) scattering amplitude grows as E^2. The s-wave amplitude grows logarithmically in high energy region just like in the standard model, and depends on $\theta_{\rm H}$ in the warped case. Thus, even if we consider only the process $W_L^+ + W_L^- \to Z_L + Z_L$, the tree-level unitarity will be violated for quite large E. It is known, however, in higher dimensional theories, the unitarity violation appears at a lower energy, by summing up all the possible final states, exhibiting the non-renormalizability. Generically the amplitude is enhanced in the warped case for $\theta_{\rm H} = \mathcal{O}(1)$. This suggests that the tree-level unitarity will be violated at a lower scale in the warped case than the flat case.

In ref. [12], three separate scales that determine the dynamics of the scattering process are introduced, i.e., the electroweak breaking scale v, the Higgs boson decay constant f_h,[7] and the KK scale $m_{\rm KK}$. In our notation, these scales are related to each other as $v = f_h\theta_{\rm H}/2$ and $f_h = \sqrt{2}/(g_5\sqrt{L}) = \sqrt{2}m_{\rm KK}/(\pi g_4)$ in the flat case, and $v = f_h \sin(\theta_{\rm H}/2)$ and $f_h \simeq 2\sqrt{k}e^{-kL}/g_5 \simeq 2m_{\rm KK}/(\pi g_4\sqrt{kL})$ in the warped case. In the terminology of ref. [12], the case of $\theta_{\rm H} \ll 1$ is referred to as the 'Higgs limit', and the case of $\theta_{\rm H} = \mathcal{O}(1)$ is as the 'Higgsless limit'. The Higgs boson unitarizes the scattering process in the former while it does not (or does only partly) in the latter.

For the purpose of estimating the scale of the unitarity violation, we should extend our analysis for the following points. We should take into account the Higgs mass, which is induced by the quantum effect, and the decay widths of the weak bosons. The latter is necessary to discuss the process: $W_L^+ + W_L^- \to W_L^+ + W_L^-$, for example. The infrared singularity for the forward scattering of this process is smeared out by taking into account the width of the W boson. Furthermore, we have to sum up all the possible final states including the KK states to discuss the unitarity. Since the SU(3) model is a toy model, we should work in a more realistic model, for example, the SO(5) $\times$ U(1) model [9–11]. In the flat spacetime, the spectrum of the latter model has a qualitatively different $\theta_{\rm H}$-dependence from the former due to the nontrivial boundary conditions of the 5D gauge fields.[8] Each mass eigenvalue is not a linear function of $\theta_{\rm H}$ (see figure 1 in ref. [11]) in contrast to the SU(3) model. This difference may affects the $\theta_{\rm H}$-independence of the scattering amplitude found in the our model. These issues will be discussed in a subsequent paper.

[7] This is the composite scale of the Higgs boson in the holographic dual picture.

[8] These boundary conditions are effectively obtained from the orbifold ones by introducing some boundary terms.

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A Bases of mode functions

Here we define bases of mode functions, following ref. [21]. The functions $C_0(y,m)$ and $S_0(y,m)$ are defined as two independent solutions to

$$\left(\frac{d}{dy}e^{-2\sigma}\frac{d}{dy}+m^2\right)f=0, \tag{A.1}$$

with initial conditions

$$\begin{aligned} C_0(0,m)&=1, & C_0'(0,m)&=0,\\ S_0(0,m)&=0, & S_0'(0,m)&=me^{-\sigma(L)}. \end{aligned} \tag{A.2}$$

For the derivation of 5D propagators in appendix B, it is convenient to define another basis functions $C_L(y,m)$ and $S_L(y,m)$ with initial conditions

$$\begin{aligned} C_L(L,m)&=1, & C_L'(L,m)&=0,\\ S_L(L,m)&=0, & S_L'(L,m)&=me^{\sigma(L)}. \end{aligned} \tag{A.3}$$

From the Wronskian relation, the above functions satisfy

$$\begin{aligned} &C_0(y,m)S_0'(y,m)-S_0(y,m)C_0'(y,m)=\\ &C_L(y,m)S_L'(y,m)-S_L(y,m)C_L'(y,m)=me^{2\sigma(y)-\sigma(L)}. \end{aligned} \tag{A.4}$$

The two bases are related to each other by

$$\begin{aligned} C_L(y,m)&=\frac{e^{-\sigma(L)}}{m}\left\{S_0'(L,m)C_0(y,m)-C_0'(L,m)S_0(y,m)\right\},\\ S_L(y,m)&=-\left\{S_0(L,m)C_0(y,m)-C_0(L,m)S_0(y,m)\right\}. \end{aligned} \tag{A.5}$$

Flat spacetime. In the flat spacetime, i.e., $\sigma(y)=0$, the basis functions are reduced to

$$\begin{aligned} C_0(y,m)&=\cos(my), & S_0(y,m)&=\sin(my),\\ C_L(y,m)&=\cos\left\{m(y-L)\right\}, & S_L(y,m)&=\sin\left\{m(y-L)\right\}. \end{aligned} \tag{A.6}$$

Randall-Sundrum spacetime. In the Randall-Sundrum spacetime, i.e., $\sigma(y)=ky$, the basis functions are written in terms of the Bessel functions as

$$\begin{aligned} C_0(y,m)&=\frac{\pi m}{2k}e^{ky}\left\{Y_0\left(\frac{m}{k}\right)J_1\left(\frac{m}{k}e^{ky}\right)-J_0\left(\frac{m}{k}\right)Y_1\left(\frac{m}{k}e^{ky}\right)\right\},\\ S_0(y,m)&=-\frac{\pi m}{2k}e^{k(y-L)}\left\{Y_1\left(\frac{m}{k}\right)J_1\left(\frac{m}{k}e^{ky}\right)-J_1\left(\frac{m}{k}\right)Y_1\left(\frac{m}{k}e^{ky}\right)\right\},\\ C_L(y,m)&=\frac{\pi m}{2k}e^{ky}\left\{Y_0\left(\frac{m}{k}e^{kL}\right)J_1\left(\frac{m}{k}e^{ky}\right)-J_0\left(\frac{m}{k}e^{kL}\right)Y_1\left(\frac{m}{k}e^{ky}\right)\right\},\\ S_L(y,m)&=-\frac{\pi m}{2k}e^{ky}\left\{Y_1\left(\frac{m}{k}e^{kL}\right)J_1\left(\frac{m}{k}e^{ky}\right)-J_1\left(\frac{m}{k}e^{kL}\right)Y_1\left(\frac{m}{k}e^{ky}\right)\right\}. \end{aligned} \tag{A.7}$$

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B Derivation of 5D propagators

Here we derive explicit forms of 5D propagators. We take the same strategy as in the appendix of ref. [14]. Since the 4D vector part A_μ and the gauge-scalar part A_y are decoupled at the quadratic level with our choice of the gauge-fixing function, the mixed components of the propagator $\langle 0|TA_\mu^\alpha(p,y)A_y^\beta(-p,y')|0\rangle$ vanish. In this section, we work in the Scherk-Schwarz basis defined by (2.9) and (2.10).

B.1 Vector propagator

The gauge index $\alpha = 1, \cdots, 8$ is decomposed into two parts as $a = 1, 2, 3', 8'$ and $\hat{a} = 4, 5, 6, 7$, according to the Z_2-parities of A_μ^α. Then the 5D propagator $iG_{\mu\nu}^{\alpha\beta}(p,y,y') \equiv \langle 0|TA_\mu^\alpha(p,y)A_\nu^\beta(-p,y')|0\rangle$ satisfies

$$\left[\left\{\partial_y^2 - 2\sigma'\partial_y - e^{2\sigma}p^2\right\}\delta_\mu{}^\nu + e^{2\sigma}\left(\frac{1}{\xi}-1\right)p_\mu p^\nu\right]G_{\nu\rho}^{\alpha\beta}(p,y,y') = e^{2\sigma}\eta_{\mu\rho}\delta^{\alpha\beta}\delta(y-y'), \quad \text{(B.1)}$$

with the boundary conditions,

$$\begin{aligned}\partial_y G_{\mu\nu}^{a\beta}(p,0,y') &= G_{\mu\nu}^{\hat{a}\beta}(p,0,y') = 0,\\ (R_\theta)^{a\gamma}\,\partial_y G_{\mu\nu}^{\gamma\beta}(p,L,y') &= (R_\theta)^{\hat{a}\gamma}\,G_{\mu\nu}^{\gamma\beta}(p,L,y') = 0,\end{aligned} \quad \text{(B.2)}$$

where a constant matrix R_θ is a rotation matrix for the indices of the adjoint representation corresponding to a transformation by $\Omega(L)$ defined in (2.10), i.e.,

$$(R_\theta)^{\alpha\beta}\,A_M^\beta = \left[\Omega^{-1}(L)A_M\Omega(L)\right]^\alpha \equiv \mathrm{tr}\left\{\lambda^\alpha\Omega^{-1}(L)A_M\Omega(L)\right\}. \quad \text{(B.3)}$$

We can decompose $G_{\mu\nu}^{\alpha\beta}(p,y,y')$ into the following two parts.

$$G_{\mu\nu}^{\alpha\beta}(p,y,y') = \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)G_{\mathrm{T}}^{\alpha\beta}(y,y',|p|) + \frac{p_\mu p_\nu}{p^2}G_{\mathrm{S}}^{\alpha\beta}(y,y',|p|), \quad \text{(B.4)}$$

where $|p| \equiv \sqrt{-p^2}$. The first and the second terms correspond to the propagators for $A_\mu^{(n)}$ and $A_{\mathrm{S}}^{(n)}$, respectively. Writing $G_{\mathrm{T}}^{\alpha\beta}(y,y',|p|)$ as

$$G_{\mathrm{T}}^{\alpha\beta}(y,y',|p|) = \vartheta(y-y')G_{\mathrm{T}>}^{\alpha\beta}(y,y',|p|) + \vartheta(y'-y)G_{\mathrm{T}<}^{\alpha\beta}(y,y',|p|), \quad \text{(B.5)}$$

the solutions to (B.1) satisfying (B.2) are given in the matrix notation for the index $\alpha = (a,\hat{a})$ by

$$\begin{aligned}G_{\mathrm{T}<}(y,y',|p|) &= \mathcal{M}_0(y,|p|)\alpha_{\mathrm{T}<}(y',|p|),\\ R_\theta G_{\mathrm{T}>}(y,y',|p|) &= \mathcal{M}_L(y,|p|)\alpha_{\mathrm{T}>}(y',|p|),\end{aligned} \quad \text{(B.6)}$$

where

$$\mathcal{M}_0 \equiv \begin{pmatrix} C_0\cdot\mathbf{1}_4 & \\ & S_0\cdot\mathbf{1}_4\end{pmatrix}, \qquad \mathcal{M}_L \equiv \begin{pmatrix} C_L\cdot\mathbf{1}_4 & \\ & S_L\cdot\mathbf{1}_4\end{pmatrix}. \quad \text{(B.7)}$$

The unknown matrix functions $\alpha_{\mathrm{T}<}(y',|p|)$ and $\alpha_{\mathrm{T}>}(y',|p|)$ are determined by imposing the following matching conditions at $y = y'$. The continuity of G_{T} at $y = y'$ leads to the condition

$$G_{\mathrm{T}<}(y,y,|p|) = G_{\mathrm{T}>}(y,y,|p|), \tag{B.8}$$

and we obtain from (B.1) the condition

$$\left\{\partial_y G_{\mathrm{T}>}(y,y',|p|) - \partial_y G_{\mathrm{T}<}(y,y',|p|)\right\}_{y'\to y} = e^{2\sigma(y)}. \tag{B.9}$$

Using these conditions, we obtain the 5D propagators as

$$\begin{aligned} G_{\mathrm{T}<}(y,y',|p|) &= e^{2\sigma(L)}\mathcal{M}_0(y,|p|)\mathcal{W}^{-1}(|p|)\mathcal{M}_L(y',|p|)R_\theta, \\ G_{\mathrm{T}>}(y,y',|p|) &= \left\{G_{\mathrm{T}<}(y',y,|p|)\right\}^t, \end{aligned} \tag{B.10}$$

where

$$\mathcal{W}(|p|) \equiv e^{-2\sigma(y)+2\sigma(L)}\left(\mathcal{M}_L' R_\theta \mathcal{M}_0 - \mathcal{M}_L R_\theta \mathcal{M}_0'\right)(y,|p|) \tag{B.11}$$

is y-independent from the Wronskian relation (A.4).

The part of the scalar modes $G_{\mathrm{S}}(y,y',|p|)$ is obtained in a similar way, and it is related to $G_{\mathrm{T}}(y,y',|p|)$ as

$$G_{\mathrm{S}}(y,y',|p|) = G_{\mathrm{T}}(y,y',|p|\,/\sqrt{\xi}). \tag{B.12}$$

B.2 Gauge-scalar propagator

Next we consider the propagators for the gauge-scalar modes. The 5D propagator $iG_{yy}^{\alpha\beta}(y,y',|p|) \equiv \langle 0|TA_y^\alpha(p,y)A_y^\beta(-p,y')|0\rangle$ satisfies

$$\left\{\xi\partial_y^2 e^{-2\sigma} - p^2\right\} G_{yy}^{\alpha\beta}(y,y',|p|) = e^{2\sigma}\delta^{\alpha\beta}\delta(y-y'), \tag{B.13}$$

with the boundary conditions,

$$G_{yy}^{a\beta}(0,y',|p|) = \partial_y\left\{e^{-2\sigma}G_{yy}^{\hat{a}\beta}\right\}(0,y',|p|) = 0, \tag{B.14}$$

$$(R_\theta)^{a\gamma}\, G_{yy}^{\gamma\beta}(L,y',|p|) = \partial_y\left\{e^{-2\sigma}\,(R_\theta)^{\hat{a}\gamma}\, G_{yy}^{\gamma\beta}\right\}(L,y',|p|) = 0. \tag{B.15}$$

These can be solved by the same manner as in the previous subsection. We find that $G_{yy}(y,y',|p|)$ is related to $G_{\mathrm{S}}(y,y',|p|)$ as

$$\begin{aligned} G_{yy<}(y,y',|p|) &= -\frac{1}{p^2}\partial_y\partial_{y'}G_{\mathrm{S}<}(y,y',|p|), \\ G_{yy>}(y,y',|p|) &= -\frac{1}{p^2}\partial_y\partial_{y'}G_{\mathrm{S}>}(y,y',|p|). \end{aligned} \tag{B.16}$$

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